5-3 Activity Draft

FM 4.3: Given graphs of f, g: Graphically (𝑓 + 𝑔)(2) = (𝑓 − 𝑔)(−1) = (𝑔 ⋅ 𝑓)(0) = � 𝑓 𝑔 � (−2) = (𝑓 ∘ 𝑔)(3) = 𝑓 + �. Similar on the following page. Also with tables.

FM 4.3: (𝑥) = 𝑥 + 2 and 𝑔(𝑥) = 1 𝑥 (𝑓 ∘ 𝑔)(𝑥) = Domain: (𝑔 ∘ 𝑓)(𝑥) = Domain:

14. (𝑥) = √4 − 𝑥 and 𝑔(𝑥) = 𝑥2 𝑓(𝑔(𝑥)) = Domain: 𝑔(𝑓(𝑥)) = Domain:

FM 4.3: Express h as a composition of two simpler functions f and g. 15. ℎ(𝑥) = (2𝑥 − 7)4 16. ℎ(𝑥) = 4 √𝑥 + 3.

FM 4.3: 4. Suppose 2 f x x bx () 3 =+− and f (2) 9 = − . Find b. APPLICATION

5. Given fx x b () 5 2 = − while g x bx () 4 = . If f g( (1) 36 ) = what is g f ( (1)) ?

6. Given that (𝑥) = 𝑐𝑐 − 3 and 𝑔(𝑥) = 𝑐𝑐 + 5 are both defined on the set of all real numbers and c is a constant, what is the value of c if (𝑓 ∘ 𝑔)(𝑥) = (𝑔 ∘ 𝑓)(𝑥) for all values of x?

S-Z 5.1 p. 361. Example 5.1.1: . Let f(x) = x 2 − 4x, g(x) = 2 − √ x + 3, and h(x) = 2x x + 1 . In numbers 1 - 3, find the indicated function value. 1. (g ◦ f)(1) 2. (f ◦ g)(1) 3. (g ◦ g)(6) In numbers 4 - 10, find and simplify the indicated composite functions. State the domain of each. 4. (g ◦ f)(x) 5. (f ◦ g)(x) 6. (g ◦ h)(x) 7. (h ◦ g)(x) 8. (h ◦ h)(x) 9. (h ◦ (g ◦ f))(x) 10. ((h ◦ g) ◦ f)(x). Some involve sign charts.

S-Z p. 366: , the order matters.4 We found that the functions f ◦ g and g ◦ f were different as were g ◦ h and h ◦ g. Thinking of functions as processes, this isn’t all that surprising. If we think of one process as putting on our socks, and the other as putting on our shoes, the order in which we do these two tasks does matter.

S-Z: 5.1 p. 367. Example 5.1.2: The surface area S of a sphere is a function of its radius r and is given by the formula S(r) = 4πr2 . Suppose the sphere is being inflated so that the radius of the sphere is increasing according to the formula r(t) = 3t 2 , where t is measured in seconds, t ≥ 0, and r is measured in inches. Find and interpret (S ◦ r)(t).

S-Z 5.1 p. 369 HW: In Exercises 13 - 24, use the given pair of functions to find and simplify expressions for the following functions and state the domain of each using interval notation. (g ◦ f)(x) (f ◦ g)(x) (f ◦ f)(x

S-Z 5.1 HW p. 370: In Exercises 31 - 40, write the given function as a composition of two or more non-identity functions. (There are several correct answers, so check your answer using function composition.) 31. p(x) = (2x + 3)3 32. P(x) = x 2 − x + 15 33. h(x) = √ 2x − 1 34. H(x) = |7 − 3x| 35. r(x) = 2 5x + 1 36. R(x) = 7 x 2 − 1 37. q(x) = |x| + 1 |x| − 1 38. Q(x) = 2x 3 + 1 x 3 − 1 39. v(x) = 2x + 1 3 − 4x 40. w(x) = x 2 x 4 +

S-Z: 5.1: HW p. 370: Let g(x) = −x, h(x) = x+ 2, j(x) = 3x and k(x) = x−4. In what order must these functions be composed with f(x) = √ x to create F(x) = 3√ −x + 2 – 4

S-Z; p. 370-1: HW 5.1: In Exercises 44 - 55, let f be the function defined by f = {(−3, 4),(−2, 2),(−1, 0),(0, 1),(1, 3),(2, 4),(3, −1)} and let g be the function defined g = {(−3, −2),(−2, 0),(−1, −4),(0, 0),(1, −3),(2, 1),(3, 2)} . Find the value if it exists. 44. (f ◦ g)(3) 45. f(g(−1)) 46. (f ◦ f)(0) 47. (f ◦ g)(−3) 48. (g ◦ f)(3) 49. g(f(−3)

S-Z: 5.1 HW p. 371: In Exercises 56 - 61, use the graphs of y = f(x) and y = g(x) below to find the function value. x y 1 2 3 4 1 2 3 4 y = f(x) x y 1 2 3 4 1 2 3 4 y = g(x) 56. (g ◦ f)(1) 57. (f ◦ g)(3) 58. (g ◦ f)(2) 59. (f ◦ g)(0) 60. (f ◦ f)(1) 61. (g ◦ g)(1)

Active Reading: 7.1.1: An oil spill is in the shape of a circular disk, beginning with a radius of 10 meters.

The oil spreads out (maintaining a circular shape), and its radius increases by 4 meters per hour. Use the slider to see the oil spill at different times.

1. After 7 hours, what is the radius of the circle? Remember that it began at 10 meters.

r= meters.

1. At that time, what will be the area of the oil spill?

A = meters2

To find the area, you first used one rule to find the radius, and then you used that radius in the formula for the area of a circle.

Active Reading 7.1.2: The radius of the circular spill started at 10 meters, and increased by 4 meters per hour.

The radius, as a function of time, is given by:

r(t)=10+4t

and the area of the circle, as a function of the radius, is:

A(r)=πr2

Substitute the expression for the radius, r(t), into the function for A(r). Then, evaluate the function A by using the expression for r(t) as the input.

Area =A(r(t))=A()=.

Note: Your answers should both have the variable t, but they should not contain the letters r or A.

Active Reading 7.1.4: Consider two functions f and g, and suppose we know the following things:

f(1)=7f(3)=11g(7)=20g(8)=3

1. What is f(g(8))?

Answer:

1. What is g(f(1))?

Answer:

Active Reading: 7.1.5: Animated function machines

Active Reading 7.1.7: Use the graph to evaluate each of the following compositions.

1. f(g(−1))=f()=
2. g(f(7))=g()=
3. g(g(−4))=g()=
4. f(f(−6))=f()=

Active Reading: 7.3.1: Suppose f(x)=3x+4 and g(x)=x2+x. Compute each of the following compositions. Remember to work inside the parentheses first.

1. f(g(5)). We can first work inside the parentheses to find the value of g(5), and then use that value as the input for f.

f(g(5))=f(52+5)=f(30)=3⋅30+4=94

1. g(f(5)). We can first work inside the parentheses to find the value of f(5), and then use that value as the input for g.

g(f(5))=g(3⋅5+4)=g(19)=192+19=380

So we found:

1. f(g(5))=
2. g(f(5))=

vs. 7.3.3: Let f(x)=3x+4 and g(x)=x2+x. Find the composite functions (f∘g) and (g∘f) by using x as the input. Remember to simplify the expression inside the parentheses first.

1. f(g(x)). We can first substitute the expression for g(x) inside the parentheses, and then use that expression as the input for f.

f(g(x))=f(x2+x)=3(x2+x)+4=3x2+3x+4

1. g(f(x)). We can first substitute the expression for f(x) inside the parentheses, and then use that expression as the input for g.

g(f(x))=g(3x+4)=(3x+4)2+(3x+4)=9x2+24x+16+3x+4=9x2+27x+20

So we found:

1. f(g(x))=
2. g(f(x))=

Active Reading: 7.3.4: Let CC represent the concentration, measured in mg⁄kg, of a blood pressure regulating drug in a patient's bloodstream as a function of time t,t, measured in hours. As time goes on, the concentration of the drug in the bloodstream decreases as the body metabolizes and/or excretes it.

At the same time, the drug is meant to help lower the patient's blood pressure P,P, measured in mm Hg. So, as the drug leaves the patient's bloodstream, the blood pressure increases.

Therefore, PP depends on C,C, and CC depends on t.t.

There is a function which represents how CC depends on t.t. We will call this function f,f, and write:

C=f(t)C=f(t)

There is another function which represents how PP depends on C.C. We will call this function g,g, and write:

P=g(C)P=g(C)

So, we would first use f(t)f(t) to find the concentration CC of the drug in the bloodstream, and then use that concentration as the input for the function g(C)g(C) in order to find the patient's blood pressure P.P.

In function notation, this would look like:

P=g(C)=g(f(t))P=g(C)=g(f(t))

That last expression, g(f(t)),g(f(t)), is a ***composite function***, and it shows that the blood pressure is really a function of t.

Active Reading: 7.3.5: The blood pressure drug *lisinopril* works best when it is consumed at an initial concentration of 5 mg/Kg, and its concentration decreases exponentially, with a half-life of 12 hours.

Therefore, the concentration can be modeled by the function

C=f(t)=5e−0.058t

where t represents the number of hours after it was consumed.

Suppose that a patient with hypertension is admitted with blood pressure at 160 mm Hg. Administering the drug at a concentration of 5 mg/Kg reduces their blood pressure to 110 mm Hg. As the drug’s concentration decreases over time, their blood pressure increases linearly, returning to 160 mm Hg when the concentration drops to 1 mg/Kg.

Therefore, the expected blood pressure can be modeled by the function:

P=g(C)=−12.5C+172.5

1. What is the concentration of the drug after 16 hours?

C=f(16)= mg/Kg

1. What is the expected blood pressure after 16 hours?

P=g(f(16))= mm Hg

Active reading 7.3.21: Suppose M=f(t) represents the number of members in the voting party, where t represents the number of years after 2000. Also, let V=g(M) represent the percent of members that we expect to vote in the upcoming election.

1. Use function notation to write an expression that represents the percent of the party that will vote when there are 200000 members.

Answer:

1. Use function notation to write an expression that represents the number of members in the voting party in the year 2013.

Answer:

1. Use function notation to write a composition that represents the percent of the party that will vote in the year 2019.

Answer:

Active Reading: 7.3.HW #2; The table below has values for functions P(x) and Q(x). Use those values to determine the values of the composition Q(P(x)).

Remember to evaluate first inside the parentheses.

Active Reading: HW #5: Suppose that f(x)=−piecewise: 5x2−5x+4 andg(x)={−4x+6x<1−31≤x<7x+3x≥7.Find the following:

(a) (f∘g)(1)=   
 (b) (g∘f)(−2)=

Active Reading 7.3: HW #6: Complete the following tables with values for the functions f, g and h given that:

**(a)** f is an odd function.

**(b)** g is an even function.

**(c)** h=g(f(x)).

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x= | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
| f(x)= | 1 | 3 | 2 | 0 |  |  |  |
| g(x)= | 1 | 3 | 2 | 0 |  |  |  |
| h(x)= |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

Active Reading 7.3: HW #14: The figures below shows the graph of f(x) in blue and the graph of g(x) in red:

Based on the figure above, graph the function f(g(x)).

Which of the graphs below labeled A-E most accurately matches your graph? (enter the corresponding letter)

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |

Active Reading. 7.3: HW #36: Your company has a fleet of vehicles that transport goods around the metropolitan area. You use the following functions:

* V(w) represents the number of miles your moving vans drive in week w
* T(w) represents the number of miles your moving trucks drive in week w
* A(m) represents the cost of fuel for driving your moving vans m miles
* B(m) represents the cost of fuel for driving your moving trucks m miles

1. Which expression represents the total cost of fuel for your moving vans and trucks in week w?

Answer:

?

A(V(w)) + B(T(w))

V(w) + T(w)

T(V(w)) + B(A(m))

V(w)T(w)

1. Which expression represents the total number of miles driven for your moving vans and trucks in week w?

Answer:

?

V(T(w)) + T(V(w))

V(A(m))

V(w)T(w)

V(w) + T(w)

Active Reading HW #37: On your popular YouTube channel, you earn a small amount of money (due to advertising) for each time your only video is viewed.

Suppose the function f(x) represents how many times your video is viewed on the xth day of the month, and g(x) represents how much you earn per view on the xth day of the month.

Finally, suppose the function h(x) represents how much you must pay in taxes for earning x dollars from your YouTube channel.

1. Which expression represents how much money you earn on the 5th day of the month?

Answer:

?

f(g(5))

g(f(5))

f(5) + g(5)

f(5)g(5)

1. Which expression represents how much you must pay in your taxes from your earnings on the 5th day of the month?

Answer:

?

h(5)

h(f(5)g(5))

h(f(5))

h(g(f(5)))

Active Reading 7.3 HW #39: Fill in the missing values in the tables below given that w(t)=v(u(t)) .

|  |  |  |  |
| --- | --- | --- | --- |
| t | u(t) | v(t) | w(t) |
| 0 | 2 | 3 |  |
| 1 |  |  | 2 |
| 2 | 1 | 1 | 4 |
| 3 |  | 2 | 0 |
| 4 | 0 | 0 |  |

Active Reading 7.3 HW #40: Let g(x)=x. Find each of the following.

|  |  |  |
| --- | --- | --- |
| (a) g(2s+6) | = |  |
| (b) 8g(9x) | = |  |
| (c) −2g(x) | = |  |
| (d) g(x) | = |  |
| (e) (g(x))4−g(x4) | = |  |
| (f) g(9x) | = |  |
| (g) g((x−1)4) | = |  |
| (h) g(x+h) | = |  |

APC: 1.6; p. 63: Activity 1.6.2. Let functions p and q be given by the graphs in Figure 1.6.4 (which are each piecewise linear - that is, parts that look like straight lines are straight lines) and let f and 1 be given by Table 1.6.3. x 0 1 2 3 4 f (x) 6 4 3 4 6 1(x) 1 3 0 4 2 Table 1.6.3: Table that defines f and 1. -3 -2 -1 1 2 3 -3 -2 -1 1 2 3 p q Figure 1.6.4: The graphs of p and q. Compute each of the following quantities or explain why they are not defined. a. p(q(0)) b. q(p(0)) c. (p ◦ p)(−1) d. ( f ◦ 1)(2) e. (1 ◦ f )(3) f. 1( f (0)) g. For what value(s) of x is f (1(x)) 4? h. For what value(s) of x is q(p(x)) 1?

APC 1.6 HW: 1. Suppose r f (t) is the radius, in centimeters, of a circle at time t minutes, and A(r) is the area, in square centimeters, of a circle of radius r centimeters. Which of the following statements best explains the meaning of the composite function A( f (t))? ⊙ The area of a circle, in square centimeters, of radius r centimeters. ⊙ The area of a circle, in square centimeters, at time t minutes. ⊙ The radius of a circle, in centimeters, at time t minutes. ⊙ The function f of the minutes and the area. ⊙ None of the above

2. A swinging pendulum is constructed from a piece of string with a weight attached to the bottom. The length of the pendulum depends on how much string is let out. Suppose L f (t) is the length, in centimeters, of the pendulum at time t minutes, and P(L) is the period, in seconds, of a pendulum of length L. Which of the following statements best explains the meaning of the composite function P( f (t))? ⊙ The period P of the pendulum, in minutes, after t minutes have elapsed. ⊙ The period P of the pendulum, in seconds, when the pendulum has length L meters. ⊙ The period P of the pendulum, in minutes, when the pendulum has length L meters. ⊙ The period P of the pendulum, in seconds, after t minutes have elapsed. ⊙ None of the above

3. The formula for the volume of a cube with side length s is V s 3 . The formula for the surface area of a cube is A 6s 2 . (a) Find the formula for the function s f (A). Which of the statements best explains the meaning of s f (A)? ⊙ The side length for a cube of surface area A ⊙ The side length for a cube of volume V ⊙ The volume of a cube of side length s ⊙ The surface area of a cube of side length s (b) If V 1(s), find a formula for 1( f (A)). Which of the statements best explains the meaning of 1( f (A))? ⊙ The volume for a cube of side length s ⊙ The surface area for a cube of side length s ⊙ The volume for a cube with surface area A ⊙ The surface area for a cube of volume V

APC: 1.6 HW #6: The number of bacteria in a refrigerated food product is given by N(T) 27T 2−97T+51, 3 < T < 33 where T is the temperature of the food. When the food is removed from the refrigerator, the temperature is given by T(t) 4t + 1.7 , where t is the time in hours.Find the composite function N(T(t)). Find the time when the bacteria count reaches 14225.

APC 1.6 HW #11: A spherical tank has radius 4 feet. The tank is initially empty and then begins to be filled in such a way that the height of the water rises at a constant rate of 0.4 feet per minute. Let V be the volume of water in the tank at a given instant, and h the depth of the water at the same instant; let t denote the time elapsed in minutes since the tank started being filled. a. Calculus can be used to show that the volume, V, is a function of the depth, h, of the water in the tank according to the function V f (h) π 3 h 2 (12 − h). (1.6.1) What is the domain of this model? Why? What is the corresponding range? b. We are given the fact that the tank is being filled in such a way that the height of the water rises at a constant rate of 0.4 feet per minute. Said differently, h is a function of t whose average rate of change is constant. What kind of function does this make h p(t)? Determine a formula for p(t). c. What are the domain and range of the function h p(t)? How is this tied to the dimensions of the tank? d. In (a) we observed that V is a function of h, and in (b) we found that h is a function 70 1.6 Composite Functions of t. Use these two facts and function composition appropriately to write V as a function of t. Call the resulting function V q(t). e. What are the domain and range of the function q? Why? f. On the provided axes, sketch accurate graphs of h p(t) and V q(t), labeling the vertical and horizontal scale on each graph appropriately. Make your graphs as precise as you can; use a computing device to assist as needed. h t V t Why do each of the two graphs have their respective shapes? Write at least one sentence to explain each graph; refer explicitly to the shape of the tank and other information given in the problem.